AH National Qualifications 2023 MODIFIED

X847/77/11

Mathematics
Paper 1 (Non-calculator)

THURSDAY, 4 MAY

9:00 AM – 10:00 AM

Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a(1-r^n)}{1-r},\ r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

Total marks — 35

Attempt ALL questions

1. Given $y = 7x\tan 2x$, find $\frac{dy}{dx}$. **2**

2. Express $\frac{3x^2 - x - 14}{(x+3)(x-1)^2}$ in partial fractions. **3**

3. A system of equations is defined by

$$\begin{aligned} x - 3y + z &= -1 \\ 3x - 2y + 4z &= 11 \\ x + 4y + 2z &= 15 \end{aligned}$$

Use Gaussian elimination to determine whether the system shows redundancy, inconsistency or has a unique solution. **3**

4. Use integration by parts to find $\int x^4 \ln x \, dx$, $x > 0$. **3**

5. Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 10x^2 + 11x - 23$$

given that $y = 2$, $\frac{dy}{dx} = 14$ when $x = 0$. **9**

6. (a) Express $z = 1 + \sqrt{3}i$ in polar form. **2**

(b) Hence, or otherwise, show that z^3 is real. **2**

MARKS

7. (a) Find an expression for $\sum_{r=1}^{n}\left(r^2+3r\right)$ in terms of n.

Express your answer in the form $\frac{1}{3}n\left(n+a\right)\left(n+b\right)$. **2**

(b) Hence, or otherwise, find $\sum_{r=11}^{20}\left(r^2+3r\right)$. **2**

8. (a) Consider the statement:

For all integers a and b, if $a < b$ then $a^2 < b^2$.

Find a counterexample to show that the statement is false. **1**

(b) Let n be an odd integer.

Prove directly that $n^2 - 1$ is divisible by 4. **2**

9. (a) State the matrix A, associated with an anti-clockwise rotation of $\frac{\pi}{2}$ radians about the origin. **1**

The matrix B is given by

$$B = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

The matrix given by AB is associated with an anti-clockwise rotation of α radians about the origin.

(b) (i) Determine AB. **1**

(ii) Find the value of α. **1**

(c) Determine the least positive integer value of n such that $(AB)^n = I$, where I is the 2×2 identity matrix. **1**

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